MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2016/2017

TMT 1181 – MATHEMATICAL TECHNIQUES II

(All sections / Groups)

2 JUNE 2017
3.00 p.m. – 5.00 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **5 pages** only including the cover page with **5 Questions** and an **Appendix**.

2. Attempt **ALL FIVE (5)** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write your answers in the Answer Booklet provided, and **start each question on a new page.**

4. Show all steps.

Answer ALL FIVE (5) Questions.

Question 1

a. Identify the ***linearity*** and ***order*** of the following differential equations:

i. $\frac{dy}{dt}+\frac{1}{3}y=t$ **[1 mark]**

ii. $5y''+y'-7y=5$ **[1 mark]**

b. A ***first order differential equation*** (DE) is given as below:

$$\frac{dy}{dx}=\frac{x^2}{5y^4-y^3}$$

i. Show that the DE is ***separable***. **[1 mark]**

ii. Solve the DE using ***separable method***. **[3 marks]**

c. A ***first order differential equation*** (DE) is given as below:

$$xy'+y=e^x$$

i. Rewrite the DE into the form of $y'+p(x)y=r(x)$. Identify $p(x)$ and $r(x)$. **[2 marks]**

ii. Find the ***integrating factor***, $u(x)=e^{\int p(x)dx}$. **[2 marks]**

Question 2

a. Given that the ***roots of the characteristic equation*** for the following ***differential equation*** (DE)

$$ay''+by'+cy=0$$

are $\lambda_1=3$ and $\lambda_2=-2$,

i. find the values of a, b and c. **[2.5 marks]**

ii. write down the ***general solution***, y, of the DE. **[0.5 mark]**

b. Given the following DE: $y''-y'-2y=x$,

i. determine whether the DE is ***homogeneous*** or ***non-homogeneous***. If it is non-homogenous, convert it to homogeneous DE. Then solve it to get y_h. **[3 marks]**

ii. find the ***particular solution***, y_p, for the DE using ***undetermined coefficients method***. **[3.5 marks]**

iii. form the ***general solution***, $y=y_h+y_p$. **[0.5 mark]**

Continued.....

Question 3

a. Determine the ***Laplace transform*** for the following functions:

i. $f(x) = \sin 3x + x$ **[2 marks]**

ii. $f(x) = x^2 e^x$ **[2 marks]**

b. Given the DE, $y'' + 2y' + 5y = 1$, show that the ***Laplace transform*** of $y(t)$ is $Y(s) = \dfrac{1}{s(s^2 + 2s + 5)}$ if $y(0) = 0$ and $y'(0) = 0$. **[3 marks]**

c. The ***partial fraction*** of $F(s) = \dfrac{1}{(s-2)(s+3)}$ is given in the form of $\dfrac{\frac{1}{5}}{(s-2)} + \dfrac{\frac{-1}{5}}{(s+3)}$.

Find the ***inverse Laplace transform,*** $L^{-1}[F(s)]$. **[3 marks]**

Question 4

Given the function $f(x) = \sqrt{x} + 2$.

a. Find $f'(x)$ and $f''(x)$.

(Hint: convert $\sqrt{x}$ into $x^{\frac{1}{2}}$ before you differentiate the function.) **[2 marks]**

b. Compute $f(1)$, $f'(1)$ and $f''(1)$. **[3 marks]**

c. Then use the results from (a) and (b) to find the ***second degree Taylor polynomial,*** $P_2(x)$, for $f(x) = \sqrt{x} + 2$ at $a = 1$. **[3 marks]**

d. Use the polynomial found in (c) to approximate $f(5)$. **[2 marks]**

Continued.....

Question 5

a. Given the following system of linear equations:

$$5x - 2y + z = 5$$
$$2x - 4y + z = -10$$
$$-2x + y + 5z = 5$$

i. Write the system above in ***matrix notation***. **[1 mark]**

ii. Represent the system above as an ***augmented matrix***. **[1 mark]**

b. Refer to the system of linear equations in (a).

i. In what condition the system in (a) can be solved using ***Jacobi method***? **[1 mark]**

ii. Does the system need to be rearranged into the form of a diagonally dominant system if it is to be solved using ***Jacobi method***? Then, solve the system using ***Jacobi method*** by performing the first **TWO iterations**. Given $x_0 = 0, y_0 = 0$ and $z_0 = 0$. **[7 marks]**

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APPENDIX

LAPLACE TRANSFORMS OF SOME ELEMENTARY FUNCTIONS

Function, $f(t)$	Laplace transform, $F(s)$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
$e^{\alpha t}$	$\frac{1}{s-\alpha}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$t^n e^{-\alpha t}$	$\frac{n!}{(s+\alpha)^{n+1}}$
$f'(t)$	$s \cdot F(s) - f(0)$
$f''(t)$	$s^2 F(s) - s f(0) - f'(0)$

General solution of the homogeneous linear second order differential equation

For Distinct real roots: $y = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$

For Repeated roots: $y = (c_1 + c_2 x) e^{\lambda x}$

For Complex conjugate roots: $y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$

Taylor polynomial degree 2

$$P_2(x) = f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!}$$

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